

MATHEMATICS

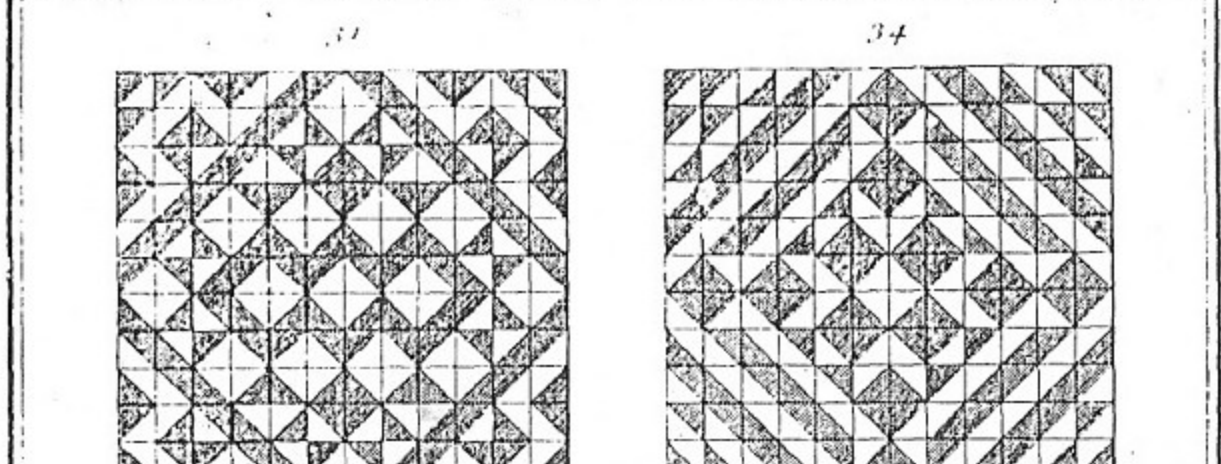
The art of the simple Truchet tile

In the previous entry of the Notebook of Scientific Culture Truchet tiling and the diamond puzzle we were talking about the design of tiling patterns made with a simple tile, the one known by the name of "Truchet tile". This is a square tile divided diagonally into two triangular areas of two different colors, for example, gray and black, like the one in the image.



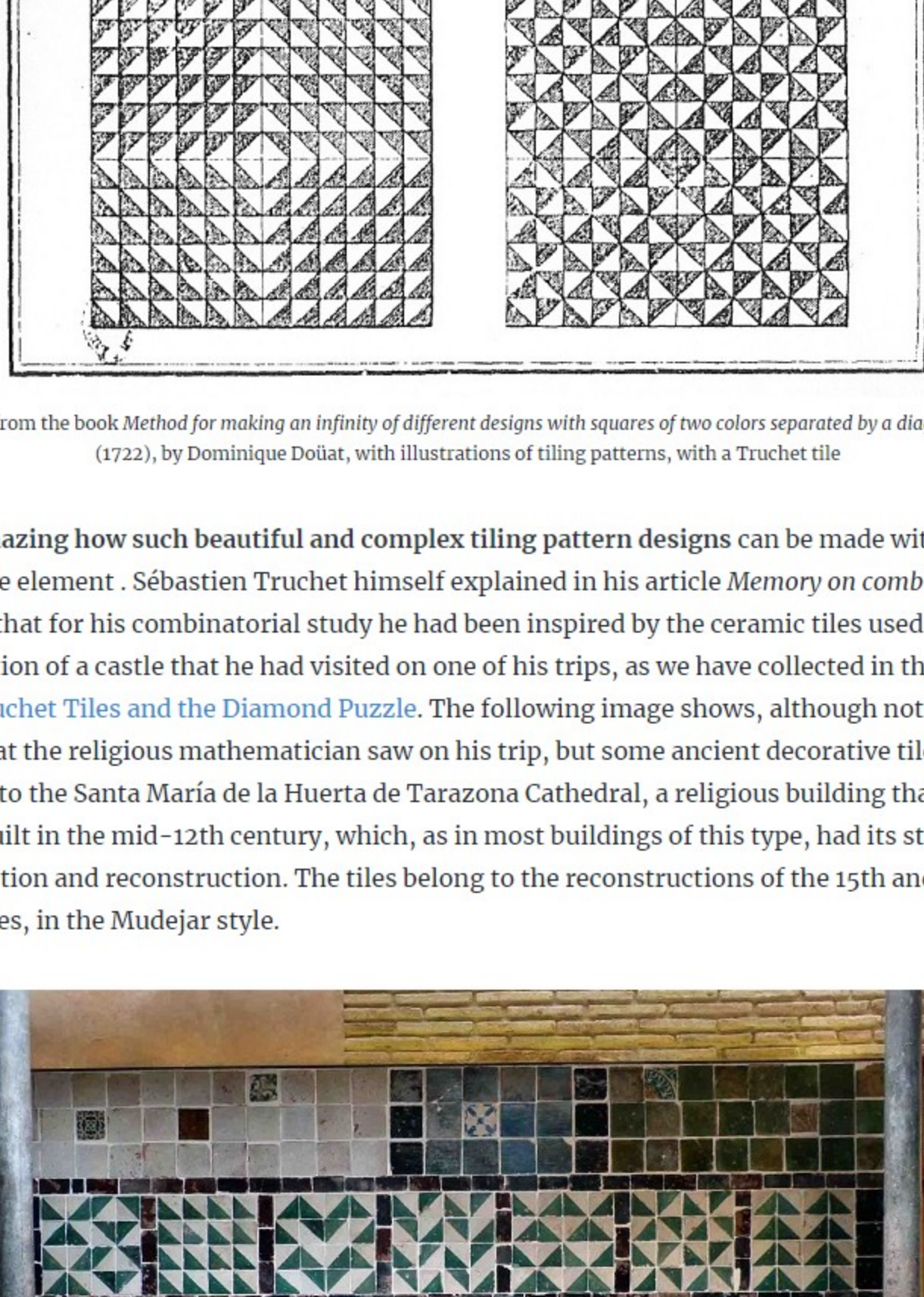
Truchet tile

In the 18th century, the French priests, interested in mathematics, Sébastien Truchet –in his article Memory of combinations (1704)– and Dominique Dolaat –in his book Method for making an infinity of different designs with squares of two colors separated by a diagonal line (1722)– took the four possible orientations of this two-color square tile, which we see in the following image, and carried out a combinatorial analysis of the possible tessellation patterns that could be built with them, which we discussed in the previous post .



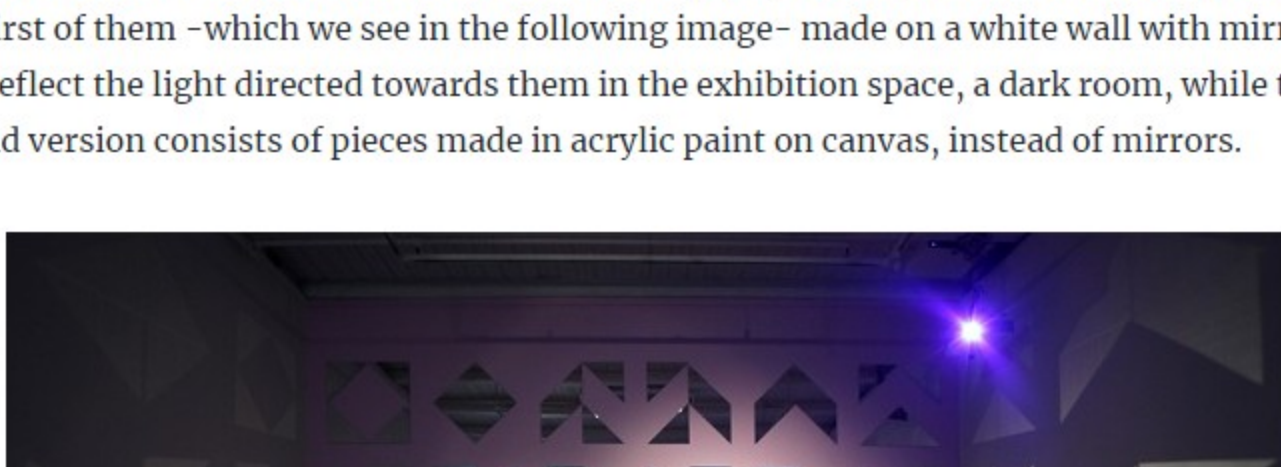
The four possible orientations of the Truchet tile, named by Dolaat as A, B, C and D

And as we mentioned then, the study of tiling or tessellation patterns, and in particular also those made with the Truchet tile, is a topic of interest both in the field of science and technology, as well as art and design . In fact, both Truchet and Dolaat not only carried out the combinatorial study of these tessellation patterns, but also took care to show beautiful designs made with this tile, such as those that appear in the following image.



A page from the book Method for making an infinity of different designs with squares of two colors separated by a diagonal line (1722), by Dominique Dolaat, with illustrations of tiling patterns, with a Truchet tile

It is amazing how such beautiful and complex tiling pattern designs can be made with such a simple element. Sébastien Truchet himself explained in his article Memory on combinations (1704) that for his combinatorial study he had been inspired by the ceramic tiles used for the decoration of a castle that he had visited on one of his trips, as we have collected in the entry The Truchet Tiles and the Diamond Puzzle. The following image shows, although not the tiles that the religious mathematician saw on his trip, but some ancient decorative tiles. They belong to the Santa María de la Huerta de Tarazona Cathedral, a religious building that began to be built in the mid-12th century, which, as in most buildings of this type, had its stages of destruction and reconstruction. The tiles belong to the reconstructions of the 15th and 16th centuries, in the Mudejar style.

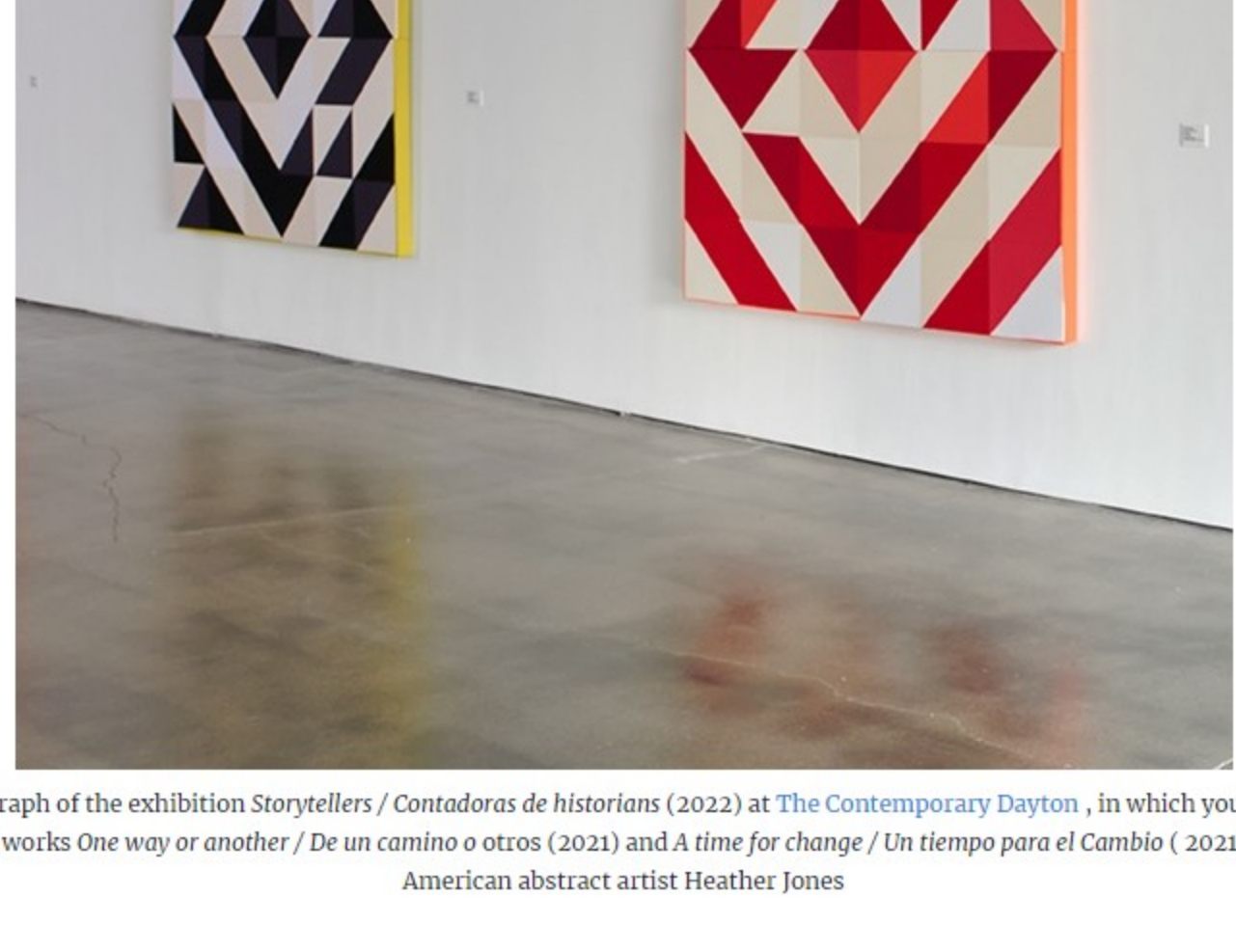


Photograph of the tiles of the Santa María de la Huerta de Tarazona Cathedral, belonging to the Symmetries I entry in the Tarazona Cathedral of the Mudejarism blog, by Angel Requena

Likewise, in the previous entry we commented on the presence of these bicolor squares in the textile art of quilts. But we can also see how some contemporary artists have used, and continue to use, these ideas in their works of art.

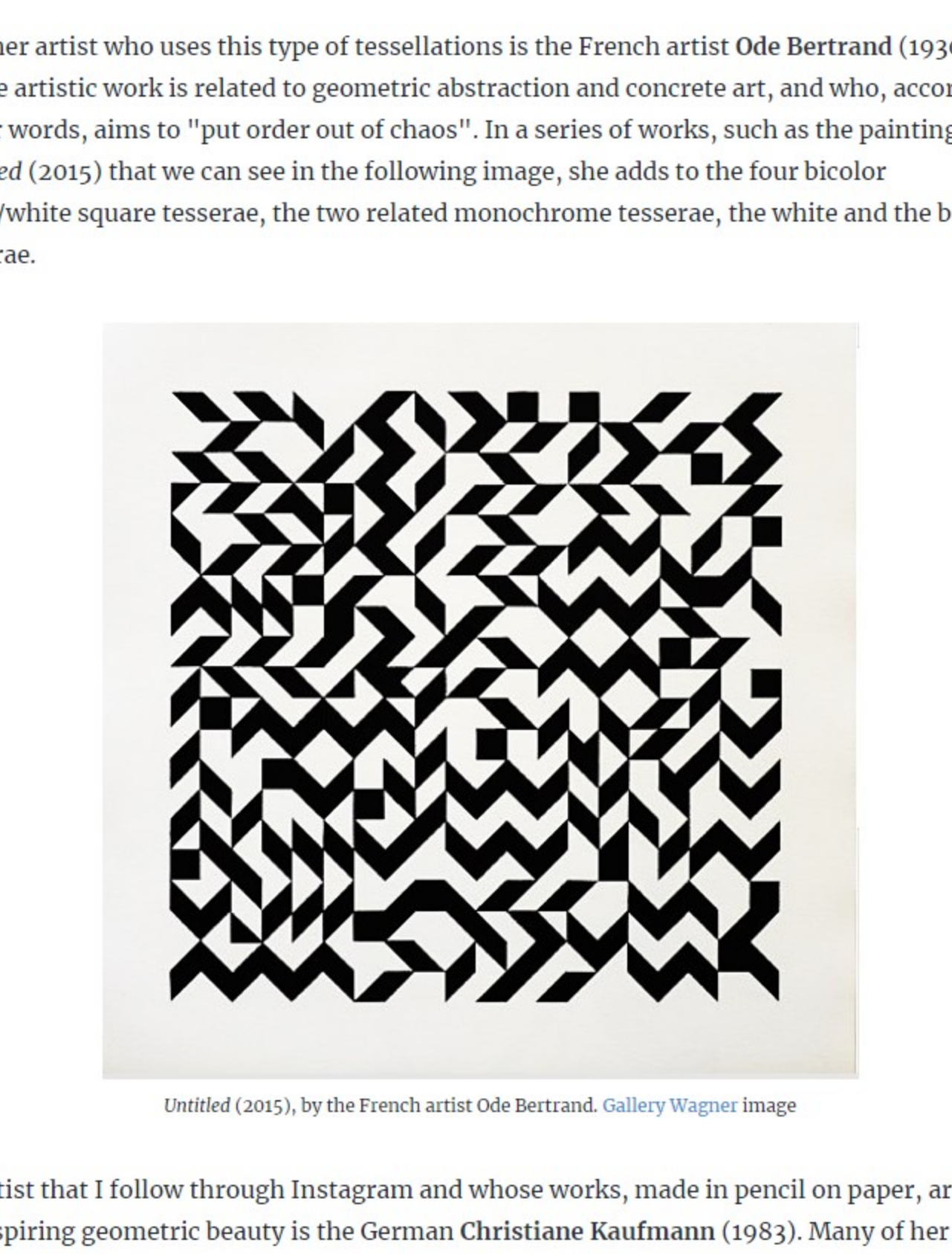
Truchet tiles in contemporary art

For example, the Norwegian artist Josefine Lyche has designed two installations with the Truchet tile tessellation patterns that are related to the diamond theorem in its 2 x 2 version (see Truchet tiling and the diamond puzzle), which they are The 2 x 2 Case (Diamond Theorem). After Steven H. Cullinane (2011) and The 2 x 2 Case (Diamond Theorem) II. After Steven H. Cullinane (2011). These works consist of the realization of the 24 designs of 2x2 square tiling that contain the four different orientations (A, B, C and D) of the two-tone square tile. The first of them –which we see in the following image– made on a white wall with mirrors that reflect the light directed towards them in the exhibition space, a dark room, while the second version consists of pieces made in acrylic paint on canvas, instead of mirrors.



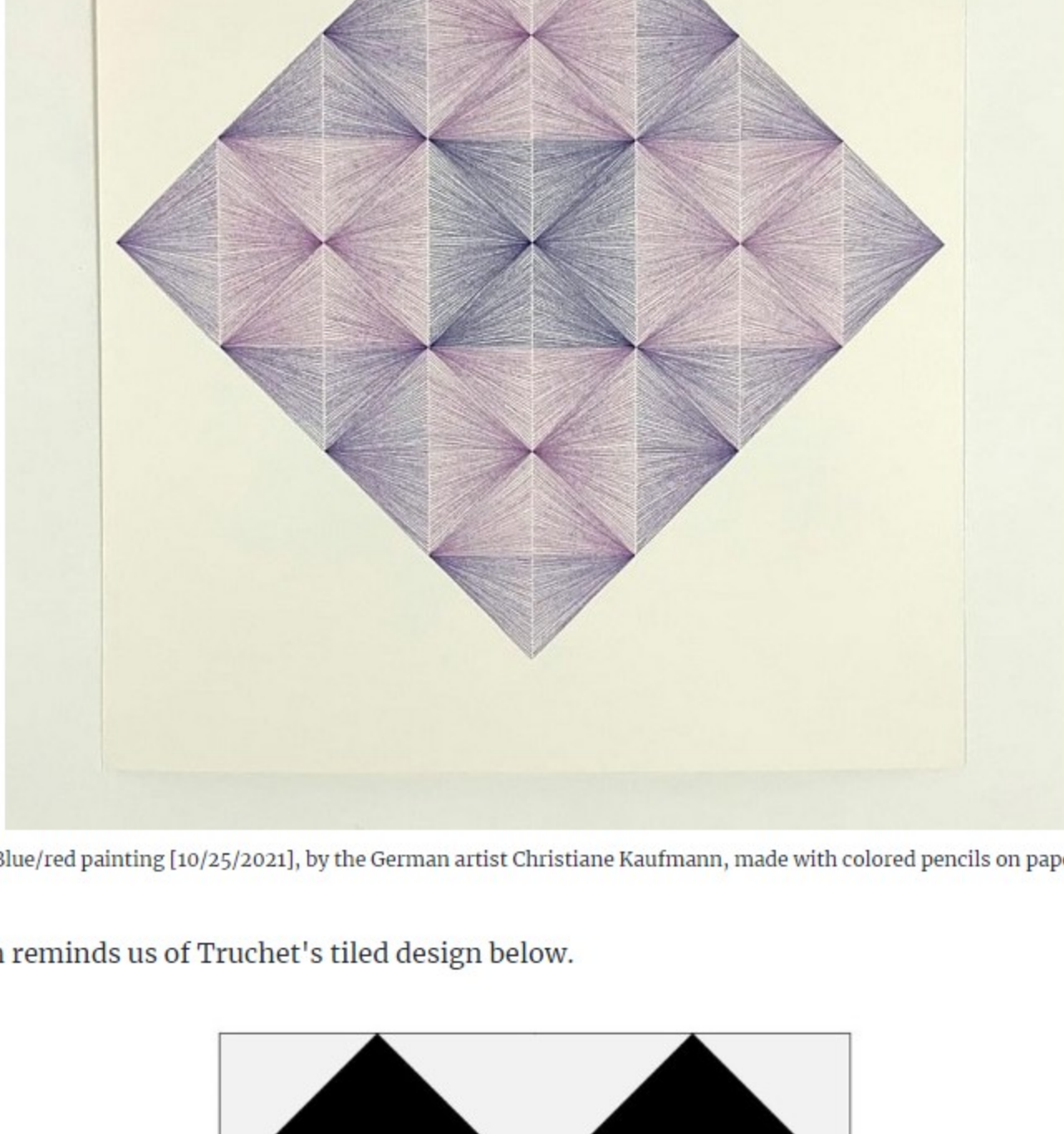
The 2 x 2 Case (Diamond Theorem). After Steven H. Cullinane (2011), by the Norwegian artist Josefine Lyche. Images from Josefine Lyche's website

American abstract artist Heather Jones, whose work is inspired by the geometric designs of traditional quilts made by women in certain regions of the United States, has created some works with this square structure divided into two colored triangles. For example, in the recent exhibition Storytellers / Contadores de historias (2022) at the contemporary art center in Dayton (Ohio), The Contemporary Dayton, dedicated to women who developed the textile art of quilts, some of these works are seen, like the ones we see in the following image: One way or another / In one way or another (2021), made of sewn cotton, highlighting the black color, and A time for change / A time for change (2021), also in sewn cotton, highlighting the color red.



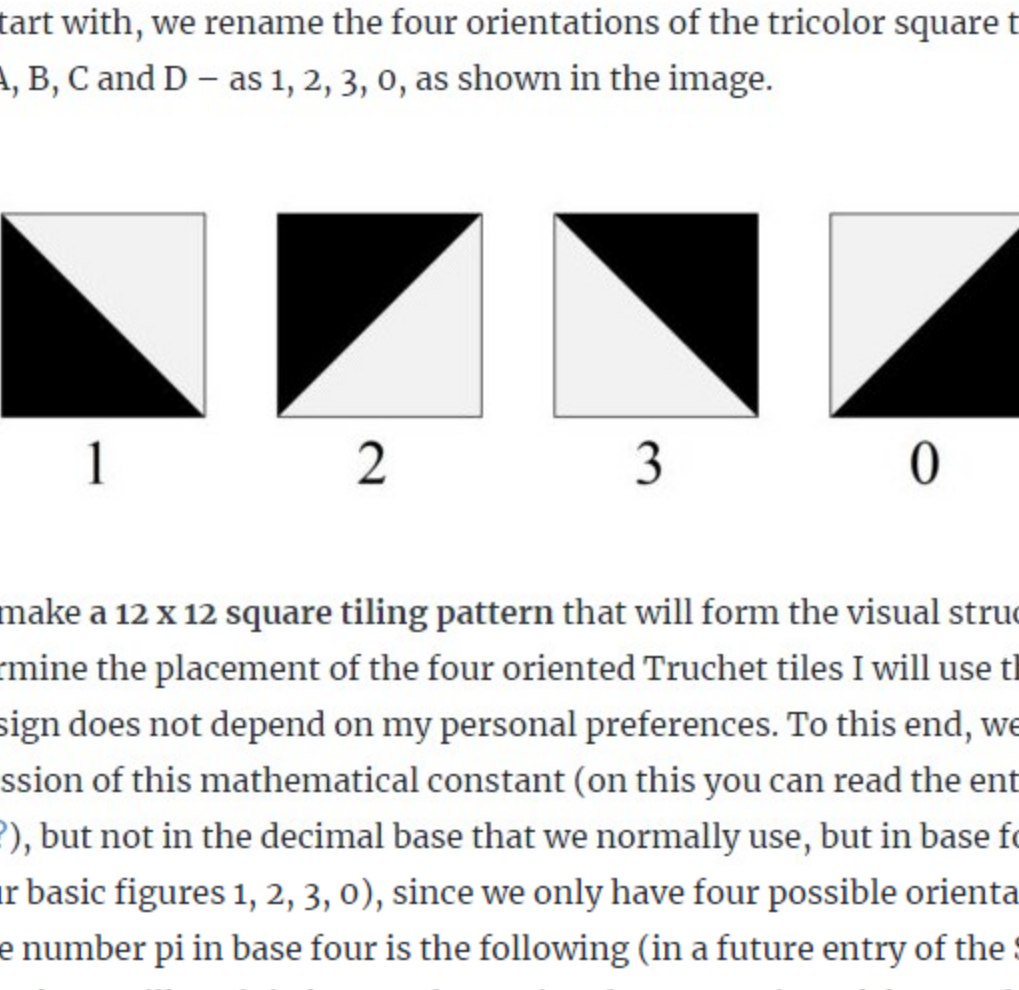
Photograph of the exhibition Storytellers / Contadores de historias (2022) at The Contemporary Dayton , in which you can see the works One way or another / In one way or another (2021) and A time for change / A time for change (2021). Collection from page image

Or also, in the recent exhibition To Hold Tender This Land / Keep this land young (2022) at the David Richard Gallery in New York it has been possible to enjoy works such as the following, entitled There was always light / Always there was light (2022).



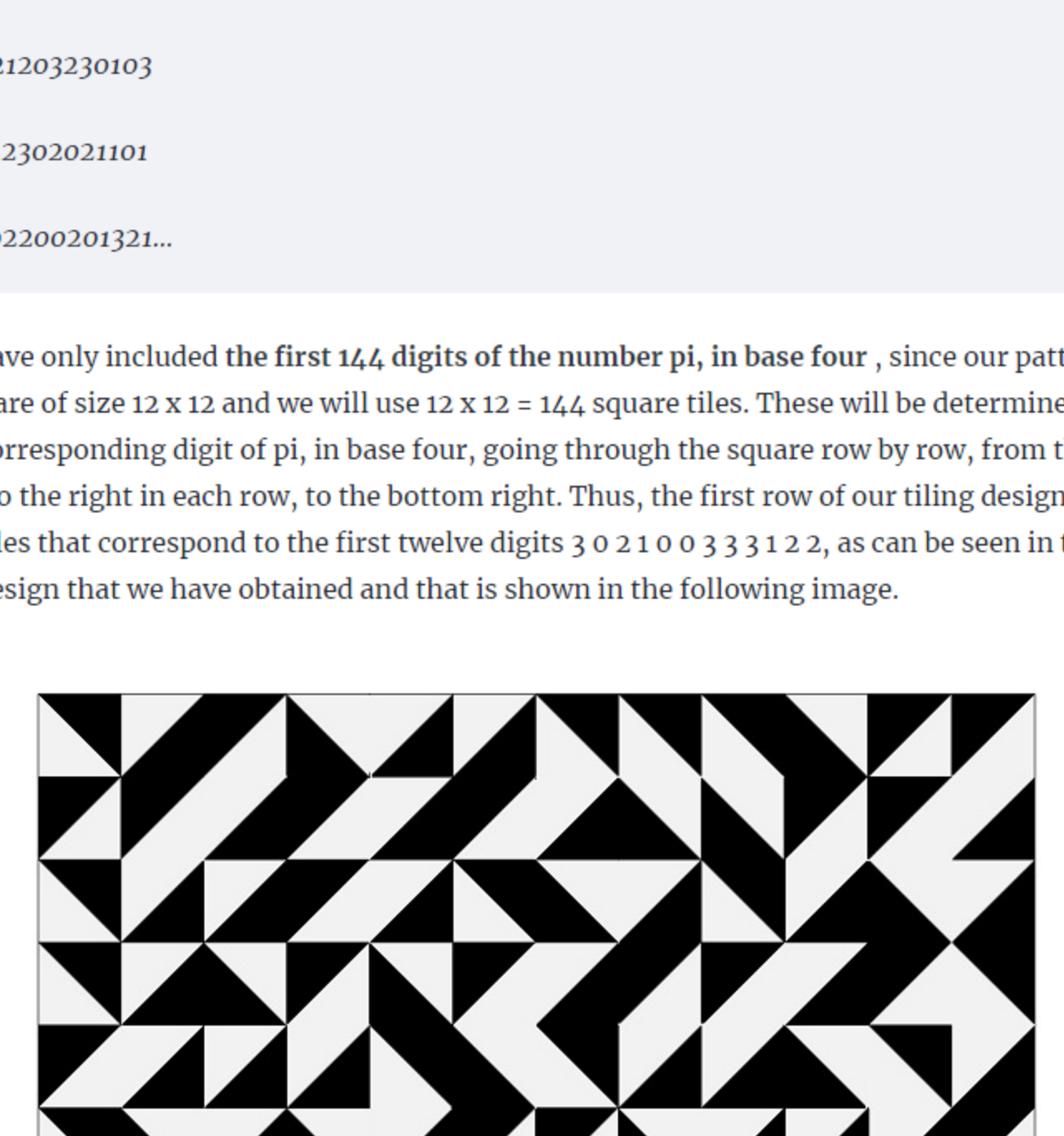
Photograph of the exhibition To Hold Tender This Land / Keep this land young (2022), by the American artist Heather Jones, at the David Richard Gallery in New York. The work on the right is There was always light / Always there was light (2022). Collection from page image

Another artist who uses this type of tessellations is the French artist Ode Bertrand (1930) whose artistic work is related to geometric abstraction and concrete art, and who, according to her words, aims to "put order out of chaos". In a series of works, such as the painting Untitled (2015) that we can see in the following image, she adds to the four bicolor black/white square tesserae, the two related monochrome tesserae, the white and the black tesserae.



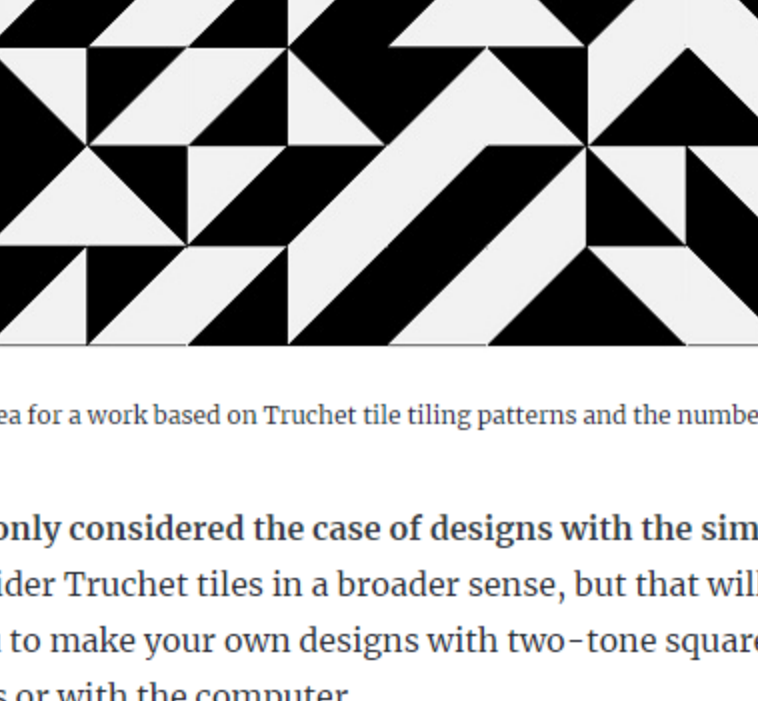
Untitled (2015), by the French artist Ode Bertrand. Gallery Wagner image

An artist that I follow through Instagram and whose works, made in pencil on paper, are of an inspiring geometric beauty is the German Christiane Kaufmann (1983). Many of her simple and beautiful creations remind us of the Truchet tile, although many of her pencil squares are single coloured. In particular, the series of works entitled blue/red , from which I highlight this piece blue/red (10/25/2021),



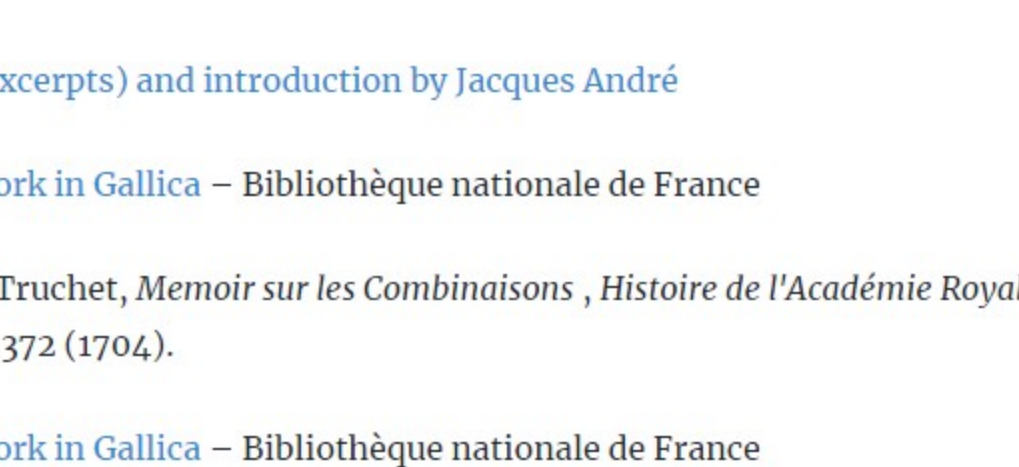
Blue/red painting [10/25/2021], by the German artist Christiane Kaufmann, made with colored pencils on paper

which reminds us of Truchet's tiled design below.



A personal artistic proposal

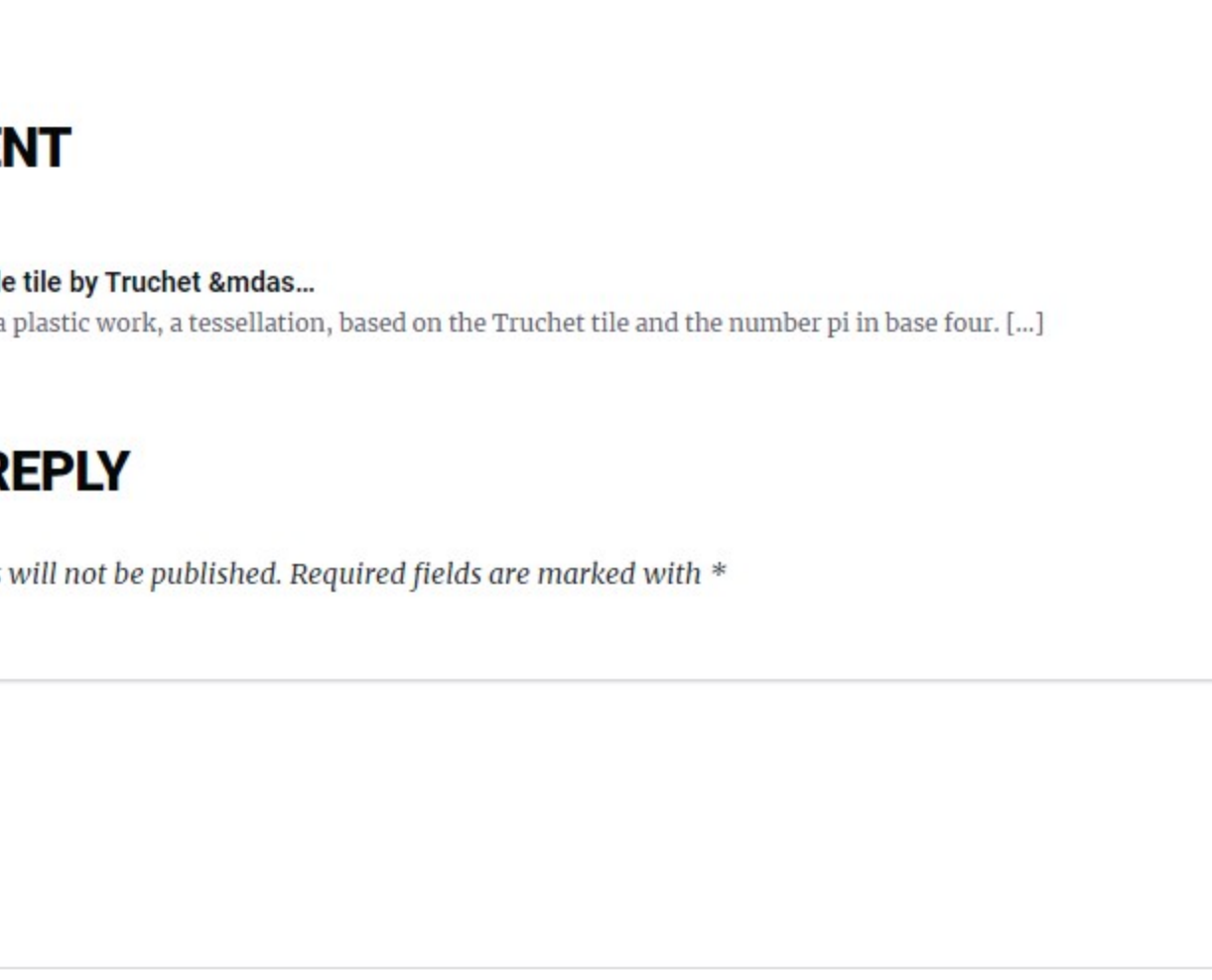
Preparing this entry of the Scientific Culture Notebook, an idea occurred to me to make a plastic work based on the Truchet tile and the number pi, whose sketch I explain in the following lines. To start with, we rename the four orientations of the tricolor square tile –previously named A, B, C and D– as 1, 2, 3, 0, as shown in the image.



The idea is to make a 12 x 12 square tiling pattern that will form the visual structure of the work. To determine the placement of the four oriented Truchet tiles I will use the number pi, that is, the design does not depend on my personal preferences. To this end, we consider the decimal expression of this mathematical constant (on this you can read the entry Is the number pi normal?), but not in the decimal base that we normally use, but in base four (in which we use the four basic figures 1, 2, 3, 0), since we only have four possible orientations. The expression of the number pi in base four is the following (in a future entry of the Scientific Culture Notebook we will explain how to determine the expression of the number pi, or any other number, in any number base, for example, four):

3.02100333122
2.2020201220
300203103010
301212022023
200031300130
310102210002
103200202023
121330303130
00020023332
221203230103
212302021101
102200201321...

We have only included the first 144 digits of the number pi, in base four, since our tile is a square of size 12 x 12 and we will use 12 x 12 = 144 square tiles. These will be determined by the corresponding digit of pi, in base four, going through the square row by row, from the top left, to the right in each row, to the bottom right. Thus, the first row of our tiling design uses the tiles that correspond to the first twelve digits 3 0 2 1 0 0 3 3 1 2 2, as can be seen in the final design that we have obtained and that is shown in the following image.



Idea for a work based on Truchet tile tiling patterns and the number pi.

In this entry we have only considered the case of designs with the simple Truchet tile, although we could consider Truchet tiles in a broader sense, but that will be another time. While I encourage you to make your own designs with two-tone squares, either with fabrics, colored pencils, paints or with the computer.

Bibliography:

1.- Cyril Stanley Smith (with translation of Truchet's text by Pauline Boucher), The Tiling Patterns of Sébastien Truchet and the Topology of Structural Hierarchy , Leonardo, vol. 20, no. 4, p. 373–385, 1987.

2.- Dominique Dolaat, Méthode pour faire une infinité de dessins différents avec des carreaux mix-partis de deux couleurs par une ligne diagonale: ou observations du Père Dominique Dolaat, Religieux Carme de la Province de Toulouse, sur un mémoire inséré dans l' Histoire de l'Académie Royale des Sciences de Paris l'année 1704, presented by the Révérend Père Sébastien Truchet religieux du même ordre, Académicien honoraire [Method for making an infinity of different designs with squares of two colors separated by a diagonal line], Paris, 1722.

* Facsimile (excerpts) and introduction by Jacques André

* Complete work in Gallica – Bibliothèque nationale de France

3.- Sébastien Truchet, Memoir sur les Combinaisons , Histoire de l'Académie Royale des Sciences de Paris , 363-372 (1704).

* Complete work in Gallica – Bibliothèque nationale de France

4.- Angel Requena, Symmetries I in the Tarazona Cathedral , from the Mudejarism blog , 2018.

5.- Josefine Lyche website

6.- Heather Jones website

7.- Christiane Kaufmann website

About the author: Raúl Ibáñez is a professor in the Department of Mathematics at the UPV/EHU and collaborator with the Chair of Scientific Culture

Information